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RUNNING ATTRIBUTION ON CREDIT PORTFOLIOS WITH DURATION TIMES SPREAD

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Introduction

Duration times spread (or DTS) is an attribution technique that has received growing attention in recent years. However, relatively few analytics vendors have implemented the algorithm in commercial software.

This paper explains what DTS attribution is and how it can add value to the investment process. We highlight its similarity to spread duration allocation, and show how to run a DTS analysis using Flametree's FIA attribution engine.

Spread duration attribution

Duration times spread (DTS) is a measure of spread risk for credit portfolios, just as modified duration is a measure of interest rate risk for rates exposures.

The core idea behind DTS is the empirical observation that the volatility of a corporate bond is proportional to the product of its spread duration and its spread, with the relationship holding over a wide range of market conditions. This product is called, naturally enough, *duration times spread*, or just DTS. Since it is usually much simpler to calculate this quantity than to work with volatilities directly, DTS has found growing usage as a measure of portfolio risk.[1]

[1] Ben Dor, A L, Dynkin, P, Houweling, J, Luuewen, E, Penninga, O, *DTS (Duration Times Spread): A New Measure of Spread Exposure in Credit Portfolios*, Lehman Brothers, June 2005

If a portfolio is managed using this risk measure, then its returns must also be measured in the same terms, and this is the motivation for the following presentation.

To understand DTS attribution, it is useful to look first at a related but simpler approach, called *spread duration allocation attribution*.

Many equity managers attempt to generate excess return by overweighting sectors in their portfolios that they believe will outperform the benchmark. If the selected sectors do in fact outperform, this larger weighting will generate a larger return for the portfolio than the benchmark. This type of overweighting decision is usually referred to as *asset allocation*, with the other being *stock selection*, where the manager tries to pick individual stocks that will outperform the benchmark, rather than particular sectors.

A fixed income manager has many additional ways to generate excess return over a benchmark, including holding higher yielding securities, adjusting the portfolio's duration to benefit from global curve movements, taking spread bets, and others. In this paper we look specifically at the effects of spread duration allocation decisions.

Just as for equity allocation, duration allocation works on the basis of overweighting particular sectors. What is overweighted, however, is not market weight but sensitivity to changes in sector spreads. This sensitivity is called the *spread duration* (or just duration, when the context is clear).

If the spread of a particular sector drops, the market value of that sector will rise, and the size of that rise will be proportional to the sector's (spread) duration. For instance, if a sector has a duration of 3 years in the benchmark but 6 years in the portfolio, that sector will contribute twice as much return to the portfolio as to the benchmark for the same drop in yield.

In general, the relationship between return R and changes in spread yield δy follows the relation

$$R = -MD \times \delta y \tag{1}$$

where MD is the spread duration.

Just as for equity portfolios, if you can pick which sectors are going to show a decrease in yield and overweight the duration of those sectors, you will generate excess returns.[2] Sector duration in duration attribution therefore has close parallels to sector weight in equity attribution.

[2] Conversely, value is also added by underweighting sectors for which spreads increase. Both the portfolio and benchmark sectors decrease in value, but the portfolio sector does not go down as much as the benchmark sector, so it has again outperfomed.

However, duration allocation attribution is somewhat more complex than equity allocation attribution. The reasons include:

1. The market weights of an equity portfolio always must sum to 100%, as must the benchmark weights. However, there is no such constraint for portfolio and benchmark duration. This gives rise to an extra degree of freedom in the analysis, since the portfolio and benchmark may well have different durations.

2. There are several ways in which the duration of a sec-

tor can be modified:

- retain the original securities but increase their market weights
- keep the market weight of the sector the same, but replace some or all of the sector's securities with others that have a higher duration
- adjust the sector's duration using a combination of the above two approaches.

Each approach will have other effects on the portfolio's overall risk, in addition to changing the sector's duration. For instance, increasing the market weight of the current securities may affect the portfolio's overall carry return, as the weight of other, possibly higher-yielding, securities will have to be decreased. Alternatively, increasing spread duration will also affect sovereign curve risk, and changes elsewhere may be necessary to keep overall durations the same.

Measuring duration allocation attribution

Duration allocation attribution generates three sources of return, compared to the two sources from equity attribution. They are labelled

- *Market direction return*
- *Duration allocation return*
- *Stock selection return*

To perform duration attribution, a weighted spread change δy^B is first calculated for the entire benchmark:

$$\delta y^B = \frac{\sum\limits_{i \in S} w_i^B MD_i^B \delta Y_i^B}{\sum\limits_{i \in S} w_i^B MD_i^B} \tag{2}$$

where w_i^B, MD_i^B and δY_i^B are the market weight, spread duration and spread change for sector i, and the sums are over all benchmark sectors.[3]

[3] For an explanation of why duration contributions are used instead of market weights in (2), see Colin, A, *Mastering Attribution in Finance*, Appendix F, FT Publishing, 2015

The *market direction return* is given by

$$R^{MD} = -(MD^P - MD^B) \times \delta y^B \tag{3}$$

where δy^B is the overall change in spread for the entire benchmark, and MD^P and MD^B are the spread durations of portfolio and benchmark respectively.

For each sector i, the *market allocation return* is given by

$$R_i^{MA} = -(w_i^P MD_i^P - w_i^B MD_i^B) \times (\delta y_i^B - \delta y^B) \tag{4}$$

where δy_i^B is the change in benchmark spread for sector i, and the other terms have the meanings defined previously.

Lastly, *security selection returns* for sector i are given by

$$R_i^{SS} = -w_i^P \times MD_i^P \times (\delta y_i^P - \delta y_i^B) \tag{5}$$

The sum of returns over all risks from equations (3), (4) and (5) reduces to

$$R_P - R_B = -\sum_{i \in S} (w_i^P MD_i^P \delta y_i^P - w_i^B MD_i^B \delta y_i^B) \tag{6}$$

which is the active return generated by all sources of spread risk, as expected.

Example of duration allocation attribution

Consider the portfolio and benchmark shown in Table 1. Here, w^P, w^B, MD^P, MD^B, δy^P, and δy^B measure market weight, spread duration and changes in spread for portfolio and benchmark respectively. We have assumed that all active return comes from market spread movements; this would be the case if, for instance, the carry and sovereign curve returns of portfolio and benchmark were the same.

The starting point for any analysis of this type is the benchmark's overall change in spread, as this is the reference point to which all portfolio yields are compared. Using equation (2), we find that for this benchmark

Sector	w^P	w^B	MD^P	MD^B	δy^P	δy^B
0-2 years	10%	15%	0.05	0.07	-1.80%	-0.25%
2-4 years	10%	40%	2.5	2.4	-1.90%	-0.20%
4-6 years	30%	15%	4.7	4.5	-0.20%	-0.30%
6-8 years	25%	20%	6.7	6.2	-0.20%	-0.23%
8-10 years	25%	10%	9.0	9.5	-0.10%	-0.50%
TOTAL	**100%**	**100%**	**5.5900**	**3.8355**		

Table 1: Illustrative weights, spread durations and spread changes

$$\delta y^B = -0.3017\%$$

There is a quite substantial difference in spread durations between portfolio (5.59 years) and portfolio (3.89 years). Overall, the portfolio's active spread duration position is 1.7 years long. Since the duration-weighted change in spread for the benchmark was a substantial -30 bps (basis points), equation (1) suggests that the active spread return should have been around -1.75 * -30 bp = 52 bps. However, the net active spread return is only 0.17%, a third of this amount. In the following analysis we use duration allocation attribution to work out where the missing spread return went.

Substituting the value of δy^B and the values from Table 1 into equations (2)-(4) gives the following results:

Bucket	Market direction	Allocation	Selection
0-2 years	-0.0017%	0.0003%	0.0078%
2-4 years	-0.2142%	0.0722%	0.4250%
4-6 years	0.2218%	-0.0013%	-0.1410%
6-8 years	0.1313%	-0.0312%	-0.0503%
8-10 years	0.3923%	0.2577%	-0.9000%
TOTAL	**0.5294%**	**0.2978%**	**-0.6585%**

Table 2: Results from duration allocation attribution analysis

This gives a much clearer picture of where return is being made and lost.

- As expected, there is a substantial return (0.53%) from market direction effects. However, this figure ignores the effects of any over- or under-weightings against individual sectors, which are calculated in the duration allocation and the duration stock selection columns.

- The largest contribution to allocation return is from the 8-10 year maturity bucket. This is generated by a combination of being 15% overweight by market value, a high duration, and a large excess spread change (-0.50%) versus the benchmark (-0.30%). Note that, by comparison, the 2-4 year bucket is underweight by 30%, but the duration is much lower and the spread change is much closer to that of the benchmark, so the overall effect is much lower.

- The selection return almost wipes out the gains made by market direction and duration allocation. The 2-4 year bucket generates a high outperformance due to the very large decrease in portfolio spreads (-1.8%) versus benchmark spreads (-0.25%). However, the opposite occurs in the 8-10 year bucket (-0.10% versus -0.50%). Although the absolute change in spreads is less, the effect of the change in spreads is amplified by almost 4 due to the sector's higher modified duration (9 years versus 2.5 years), and this generates a large underperformance in the longer-dated bucket. So the portfolio drastically underperformed the benchmark in the longest-dated bucket due to differences in individual security spread changes. Quite possibly the bonds in this bucket did not follow the rest of the market in terms of falling spreads, perhaps because of credit concerns or other security-specific issues.

Measuring DTS attribution

Assuming that market spread S is available, Equation (1) can be rewritten

$$R = -MD \times S \times \frac{\delta y}{S} \tag{7}$$

or just

$$R = -DTS \times \overline{\delta y} \tag{8}$$

where

$$DTS = MD \times S \tag{9}$$

and

$$\overline{\delta y} = \frac{\delta y}{S} \tag{10}$$

where MD is spread duration, δS is change in spread, DTS is duration times spread, and $\overline{\delta y}$ is the relative (or percentage) change in spread, rather than the absolute change used previously. Using this transformation allows equations (2), (3) and (4) to be rewritten as follows.

Firstly we calculate the change in relative spread $\overline{\delta y^B}$ for the benchmark, just as we calculated the absolute change in spread δy^B in the earlier example. This is given by

$$\overline{\delta y^B} = \frac{\sum\limits_{i \in S} w_i^B MD_i^B \overline{\delta y_i^B}}{\sum\limits_{i \in S} w_i^B MD_i^B} \tag{11}$$

The *DTS market direction return* is given by

$$R^{MD} = -(DTS^P - DTS^B) \times \overline{\delta y^B} \tag{12}$$

where $\overline{\delta y^B}$ is the overall change in yield for the entire benchmark, and MD^P and MD^B are the spread durations for portfolio and benchmark respectively.

For each sector i, the *DTS allocation return* is given by

$$R_i^{MA} = -(w_i^P DTS_i^P - w_i^B DTS_i^B) \times (\overline{\delta y_i^B} - \overline{\delta y^B}) \tag{13}$$

where δy_i^B is the change in benchmark yield for sector i, and DTS_i^P and DTS_i^B are the values of DTS for sector i in portfolio and benchmark respectively.

Lastly, the *DTS selection return* for sector i is given by

$$R_i^{SS} = -w_i^P \times DTS_i^P \times (\overline{\delta y_i^P} - \overline{\delta y_i^B}) \tag{14}$$

When spreads are very low, the relationship between spreads and volatility may break down. To ensure that Equation (10) does not generate excessive returns due to small spreads, it may be useful to define a DTS *floor* to prevent this occurring.

Example of DTS attribution

To illustrate DTS attribution, we use the same numbers as in the previous example, but provide additional data on absolute spreads S_i^P and S_i^B in order to calculate DTS. Here the starting point is Table 1, with the extra data required provided in Table 3. For convenience, this table also displays calculated values of DTS_i and $\overline{\delta y_i}$.

Bucket	s^P	s^B	DTS^P	DTS^B	$\overline{\delta y^P}$	$\overline{\delta y^B}$
0-2 years	0.032	0.0475	0.0016	0.0033	-0.5625	-0.0526
2-4 years	0.031	0.048	0.0775	0.1152	-0.6129	-0.0417
4-6 years	0.048	0.047	0.2256	0.2115	-0.0417	-0.0638
6-8 years	0.048	0.0477	0.3216	0.2957	-0.0417	-0.0482
8-10 years	0.048	0.049	0.4320	0.4655	-0.0208	-0.1020

Table 3: Absolute spreads and values of DTS

As before, the starting point is knowing the overall change in spread in the benchmark. However, here we are using DTS, rather than δS. Using equation (11) gives a value of

$$DTS^B = -6.2699\%$$

Substituting this value and the values from Tables 1 and 4 into equations (12) to (14) gives

Bucket	DTS market direction	DTS allocation	DTS selection
0-2 years	-0.0021%	0.0003%	0.0082%
2-4 years	-0.2402%	0.8050%	0.4427%
4-6 years	0.2253%	0.0042%	-0.1500%
6-8 years	0.1332%	-0.0307%	-0.5267%
8-10 years	0.3851%	0.2419%	-0.8770%
TOTAL	0.5013%	0.2962%	-0.6288%

Table 4: : Results from DTS allocation attribution analysis

Note that the overall sum of changes is the same as for the duration allocation analysis. Although DTS and duration allocation break down returns in different ways, the overall active return must always be the same.

This analysis shows particularly clearly the links between the two types of analysis. While some of the signs of the returns differ (for instance, allocation return for the 4-6 year bucket is negative, but positive for DTS allocation) the relative contributions from the three measured effects are roughly similar. Just as before, substantial returns are made from market direction and allocation returns, but heavy losses are generated at the selection level.

An Excel spreadsheet showing both sets of calculations described in this paper is available from the author.

Using DTS in Flametree FIA

DTS analysis is available in Release 1.13 and later. Please contact Technical Support if you would like to upgrade.

To run a DTS analysis in FIA, follow these steps:

- Enter the names of the sector(s) you want to use for attribution. For instance, if you have overweighted some maturity buckets and underweighted others, enter 'Maturity' into the 'Allocation' sectors field in section 'Spread effects' on the Attribution tab. The 'Spread return' section of the attribution tree on the right side of the GUI will then update to show a spread market direction effect and a spread maturity allocation effect as well as the original spread effect.
- Click the 'DTS' check box in the same section. The attribution tree will update to show *DTS attribution* rather than *Spread*, and your attribution reports will now use DTS for spread attribution.
- Run the analysis as usual.

Biography

ANDREW COLIN is founder of Flametree Technologies, a company that provides innovative performance and attribution software to fund managers of all sizes. He was previously Head of Fixed Income Research at StatPro Ltd, and has held positions in finance, academia and defence in the UK and Australia.

Andrew holds a PhD in applied mathematics from the University of St Andrews. He is a Fellow of the Institute of Mathematics and its Applications, and holds Chartered Mathematician (C.Math) accreditation. He is also Adjunct Professor in the School of Business at the University of Tasmania.

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